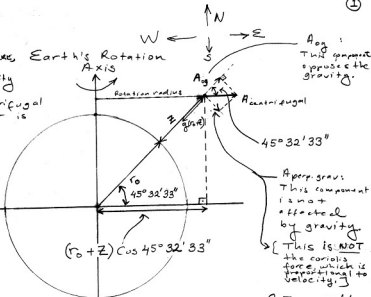


①

Acceleration of Gravity

$g_{eff}(\vec{r})$ is the effective acceleration toward the center of the Earth.

Equatorial
plane



$$A_{\text{centrifugal}} = \omega^2 (r_0 + z) \cos 45^\circ 32' 33''; \omega = \frac{2\pi}{86,164.1} \text{ rad/sec}$$

$A_{\text{og}} = A_{\text{centrifugal}} \cdot \cos 45^\circ 32' 33''$ Then;

$$A_{02} = w^2 (r_0 + z) \cos^2 45^\circ 32' 33''; \quad S_0$$

$$g_z(z) = g(r_0 + z) - \omega^2 r_0 \cos^2 45^\circ 32' 33'' - \omega^2 z \cos^2 45^\circ 32' 33''$$

(2)

$$g(r_0 + z) = g_{r_0} \left(\frac{r_0}{r_0 + z} \right)^2, \text{ as}$$

$$g(r_0 + z) = \frac{G M_E}{(r_0 + z)^2} = \underbrace{\frac{G M_E}{r_0^2}}_{g_{r_0}} \cdot \left(\frac{r_0}{r_0 + z} \right)^2.$$

M_E = Mass of the Earth
 G = Gravitational Const.

$$g_{\text{eff}}(z) = g_{r_0} \left(\frac{r_0}{r_0 + z} \right)^2 - \omega^2 r_0 \cos^2 45^\circ 32' 33'' \\ - \omega^2 z \cos^2 45^\circ 32' 33''$$

Note that; at $z=0$;

$$g_{\text{eff}}(0) = \underbrace{g_{r_0} - \omega^2 r_0 \cos^2 45^\circ 32' 33''}_{g_0 \text{ in the standard 1976.}}. \text{ Hence;}$$

$$g_{r_0} = g_0 + \omega^2 r_0 \cos^2 45^\circ 32' 33''; \text{ Then;}$$

$$g_{\text{eff}}(z) = g_0 \left(\frac{r_0}{r_0 + z} \right)^2 + \omega^2 r_0 \cos^2 45^\circ 32' 33'' \left[\left(\frac{r_0}{r_0 + z} \right)^2 - 1 \right] \\ - \omega^2 z \cos^2 45^\circ 32' 33''$$

This is the correct formula.

It can be put in a nice form;

Consider:

$$\left(\frac{r_0}{r_0+z}\right)^2 - 1 = \frac{r_0^2 - r_0^2 - 2r_0z - z^2}{(r_0+z)^2} = -\frac{z(z+2r_0)}{(r_0+z)^2}$$

$$= -2\frac{z}{r_0} \left(\frac{1+\frac{z}{2r_0}}{(1+\frac{z}{r_0})^2}\right); \quad \text{Call } \cos^2 45^\circ 32' 33'' = D;$$

$$D = 0.490532154698 \quad \text{Then;}$$

$$g_{\text{eff}}(z) = g_0 \left(\frac{r_0}{r_0+z}\right)^2 - \omega^2 D z \left\{ 2 \left(\frac{1+\frac{z}{2r_0}}{(1+\frac{z}{r_0})^2}\right) + 1 \right\}$$

$$= g_0 \left(\frac{1}{1+z/r_0}\right)^2 - \omega^2 D z \left\{ \frac{2 + \frac{z}{r_0} + 1 + \frac{2z}{r_0} + \frac{z^2}{r_0^2}}{(1+z/r_0)^2} \right\}$$

$$g_{\text{eff}}(z) = \frac{g_0 - \omega^2 D z \left\{ 3 + \frac{3z}{r_0} + \frac{z^2}{r_0^2} \right\}}{(1+z/r_0)^2} \quad \text{or,}$$

$$g_{\text{eff}}(z) = \frac{g_0 - \omega^2 D \{ 1 + (1+z/r_0) + (1+z/r_0)^2 \} z}{(1+z/r_0)^2}$$

yo's implementation!